Johann Bernoulli

Born in Basel, Swiss Confederacy, on July 27th, 1667, seventeen years before Leibniz would publish his earth-shattering paper on calculus, Johann was the younger brother and lifelong rival of his equally famed mathematician brother, Jakob. (Jakob introduced the term integral in 1690; Johann claimed it was his coinage.) His father wanted him to be a merchant, but he was excited by mathematics. His first work in mathematics was *Solutio problematis funicularii*, which appeared in *Acta eruditorum* (the same journal Leibniz’s paper appeared in) in 1690.

Johann made fun of his brother frequently. When Jakob though he had correctly described the shape of a sail, pointing out that in different circumstances it might take different shapes, Johann seems to think he is claiming that the curve which describes the shape of one sail is made up of parts of other curves [Peiffer]. When Jakob shared with Johann the differential equation that yields the answer, Johann solved it and said: “Once more he forces me to complete the solution that he has begun and developed until this equation, after which he apparently gave up” (*Journal des savants*, 1692, p. 189).

It seems likely that Johann did not want to be outshone by his brother, or always considered as secondary to the great Jakob. In a letter to Leibniz, Jakob claims to have been the first to understand Leibniz’s calculus. When Jakob found a formula for curvature, Johann printed it on cards and passed them out as his business card! Jakob then translated his formula into polar coordinates and announced it as “*inconnu même de mon frère*” – “unknown even by my brother.” [Peiffer]

Johann wrote to L’Hôpital on January 12, 1695, speaking of his brother: “He is a misanthropist in general and does not even spare his own brother...He is filled with rage, hate, envy and jealousy against me.” In a later letter: “You would not believe how much this brother, unworthy of the name, hates me, persecutes me and tries to destroy me.” [Peiffer]

In Paris through most of 1691, Johann taught calculus to the Marquis de L’Hôpital, who wrote the first book on the subject in 1696, *Analyse des infiniments petits*.

Johann became a Doctor of Medicine in 1694 when he applied differential calculus to muscular contractions.

Johann proposed a problem, called the brachistochrone problem, in *Acta eruditorum* in June 1696: what is the curve of quickest descent of a body moving from a higher point to a lower point in a plane; or, from Wolfram Mathworld, “Find the shape of the curve down which a bead sliding from rest and accelerated by gravity will slip (without friction) from one point to another in the least time. The term derives from the Greek βραχιστοσ (brachistos) "the shortest" and χρόνοσ (chronos) “time, delay.” ”

Five solutions were published in the magazine almost a whole year later, in May, 1697. One of them appeared anonymously.

At this time, Newton was retired, and he was working in the Royal Mint. He got home one day and saw the problem in a letter from Johann. It had been a long day. He was tired. But he sat down and solved the problem. Right then and there, in one night. He sent his answer to the magazine anonymously. It had taken Bernoulli two weeks to solve the problem.

When Bernoulli read the anonymous solution, he knew right away that Isaac Newton had come up with it. How did he know? Well, what he said was,

“I know the lion by his paw.”

Johann arrived at a solution by thinking about a beam of light. He knew, according to Fermat’s Principle, that whatever path between two points requires the least time, is the path that a beam of light will always follow. He imagined how a beam of light would move if affected by gravity.

Leibniz took one week to solve the problem.

In 1700, Johann’s son Daniel was born. Daniel would also become a mathematician, and a friend of the great Euler. According to E.T. Bell [Men of Mathematics] Daniel was “kicked out of the house” for winning a
prize which his father had also competed for. Johann also stole one of his son’s papers, changed the name and date, and submitted it as his own work. Nikolaus and Johann II were the other sons of Johann Bernoulli, and they too were fine mathematicians.

One of the most important activities of Johann was his fierce partisanship of Leibniz in the calculus priority dispute. “Johann saw himself as Horatio, bravely defending Leibnizian calculus from the arrogant, misguided English.” [Tent] Johann fostered the idea that Newton had plagiarized Leibniz, and in a letter of 1713 he attacked Newton’s character – he later denied having written the letter. Johann encouraged Leibniz to publish “challenge problems” which he felt the Newtonian mathematicians would be unable to solve with what he believed were their inferior methods. “Newton would, as you know, find himself in difficulties.” (Hall, p. 216). The French Academy of Sciences took Leibniz’s side, printing an attack by Bernoulli on Newton’s *Principia* without allowing a response. (Hall, p. 214). After Leibniz’s death, Bernoulli did try to mend fences with Newton, writing “I desire nothing so much as to live in good fellowship with him, and to find an opportunity of showing him how much I value his rare merits, indeed I never speak of him save with much praise.” (Hall, p. 238).

One of Johann’s greatest contributions was what he referred to as “exponential calculus” (now considered just a part of calculus) which contained the rule \( \frac{d}{dx} \ln(u) = \frac{du}{u} \). However, Johann also believed that logs of negative numbers existed and were real numbers. “He drew this remarkable, counterintuitive conclusion by applying what we would call the chain rule in taking the derivative of \( \ln(-x) \).” [Bedard]

Another tremendously important contribution of Johann Bernoulli to mathematics is an indirect one: he was the tutor of the great Leonhard Euler. Euler honored Bernoulli, calling him “illustrious.”

Bedard, Paul: “Euler and the Bernoullis: Learning by Teaching” *MAA Convergence* August 2015.


Questions

1. List 5 noted mathematicians from the Bernoulli family.

2. What seems to have been the attitude of Bernoulli towards the intellectual property of others?

3. What is the name (no longer used) of the branch of calculus which Johann Bernoulli was a pioneer in?

4. What error did Johann make about logarithms?

5. Applying the chain rule and ignoring the domain issues, find the (nonexistent) derivative of \( y = \ln(-x), \ x > 0 \)

6. How did Johann solve the brachistochrone problem?

7. Johann said of Newton, “I never speak of him save with much praise.” What is one quote which belies this claim?

8. How did Johann hope to humiliate British mathematicians?
9. Other than Euler, name a famous student of Johann Bernoulli.

10. Who introduced the term *integral*?

11. What did Johann print on his business cards?

12. How long did it take Leibniz to solve the brachistochrone problem?

13. How long did it take Newton to solve the brachistochrone problem?

14. What earned Johann Bernoulli a doctorate in medicine?