...It appears to me that the examples of mathematics and natural philosophy, which, as we have seen, were brought into their present condition by a sudden revolution, are sufficiently remarkable to fix our attention on the essential circumstances of the change which has proved so advantageous to them, and to induce us to make the experiment of imitating them, so far as the analogy which, as rational sciences, they bear to metaphysics may permit. It has hitherto been assumed that our cognition must conform to the objects; but all attempts to ascertain anything about these objects a priori, by means of conceptions, and thus to extend the range of our knowledge, have been rendered abortive by this assumption. Let us then make the experiment whether we may not be more successful in metaphysics, if we assume that the objects must conform to our cognition. This appears, at all events, to accord better with the possibility of our gaining the end we have in view, that is to say, of arriving at the cognition of objects a priori, of determining something with respect to these objects, before they are given to us. We here propose to do just what Copernicus did in attempting to explain the celestial movements. When he found that he could make no progress by assuming that all the heavenly bodies revolved round the
spectator, he reversed the process, and tried the experiment of assuming that the spectator revolved, while the stars remained at rest. We may make the same experiment with regard to the intuition of objects. If the intuition must conform to the nature of the objects, I do not see how we can know anything of them a priori. If, on the other hand, the object conforms to the nature of our faculty of intuition, I can then easily conceive the possibility of such an a priori knowledge...

Introduction

I. Of the difference between Pure and Empirical Knowledge
That all our knowledge begins with experience there can be no doubt. For how is it possible that the faculty of cognition should be awakened into exercise otherwise than by means of objects which affect our senses, and partly of themselves produce representations, partly rouse our powers of understanding into activity, to compare to connect, or to separate these, and so to convert the raw material of our sensuous impressions into a knowledge of objects, which is called experience? In respect of time, therefore, no knowledge of ours is antecedent to experience, but begins with it.

But, though all our knowledge begins with experience, it by no means follows that all arises out of experience. For, on the contrary, it is quite possible that our empirical knowledge is a compound of that which we receive through impressions, and that which the faculty of cognition supplies from itself (sensuous impressions giving merely the occasion), an addition which we cannot distinguish from the original element given by sense, till long practice has made us attentive to, and skilful in separating it. It is, therefore, a question which requires close investigation, and not to be answered at first sight, whether there exists a knowledge altogether independent of experience, and even of all sensuous impressions? Knowledge of this kind is called a priori, in contradistinction to empirical knowledge, which has its sources a posteriori, that is, in experience.

But the expression, “a priori,” is not as yet definite enough adequately to indicate the whole meaning of the question above started. For, in speaking of knowledge which has its sources in experience, we are wont to say, that this or that may be known a priori, because we do not derive this knowledge immediately from experience, but from a general rule, which, however, we have itself borrowed from experience. Thus, if a man undermined his house, we say, “he might know a priori that it would have fallen;” that is, he needed not to have waited for the experience that it did actually fall. But still, a priori, he could not know even this much. For, that bodies are heavy, and, consequently, that they fall when their supports are taken away, must have been known to him previously, by means of experience.

By the term “knowledge a priori,” therefore, we shall in the sequel understand, not such as is independent of this or that kind of experience, but such as is absolutely so of all experience. Opposed to this is empirical knowledge, or that which is possible only a posteriori, that is, through experience. Knowledge a priori is either pure or impure. Pure knowledge a priori is that with which no empirical element is mixed up. For example, the proposition, “Every change has a cause,” is a proposition a priori, but impure, because change is a conception which can only be derived from experience.

II. The Human Intellect, even in an Unphilosophical State, is in Possession of Certain Cognitions “a priori”.
The question now is as to a criterion, by which we may securely distinguish a pure from an empirical cognition. Experience no doubt teaches us that this or that object is constituted in such and such a manner, but not that it could not possibly exist otherwise. Now, in the first place, if we have a proposition which contains the idea of necessity in its very conception, it is priori. If, moreover, it is not
derived from any other proposition, unless from one equally involving the idea of necessity, it is absolutely a priori. Secondly, an empirical judgement never exhibits strict and absolute, but only assumed and comparative universality (by induction); therefore, the most we can say is—so far as we have hitherto observed, there is no exception to this or that rule. If, on the other hand, a judgement carries with it strict and absolute universality, that is, admits of no possible exception, it is not derived from experience, but is valid absolutely a priori.

Empirical universality is, therefore, only an arbitrary extension of validity, from that which may be predicated of a proposition valid in most cases, to that which is asserted of a proposition which holds good in all; as, for example, in the affirmation, “All bodies are heavy.” When, on the contrary, strict universality characterizes a judgement, it necessarily indicates another peculiar source of knowledge, namely, a faculty of cognition a priori. Necessity and strict universality, therefore, are infallible tests for distinguishing pure from empirical knowledge, and are inseparably connected with each other. But as in the use of these criteria the empirical limitation is sometimes more easily detected than the contingency of the judgement, or the unlimited universality which we attach to a judgement is often a more convincing proof than its necessity, it may be advisable to use the criteria separately, each being by itself infallible.

Now, that in the sphere of human cognition we have judgements which are necessary, and in the strictest sense universal, consequently pure a priori, it will be an easy matter to show. If we desire an example from the sciences, we need only take any proposition in mathematics. If we cast our eyes upon the commonest operations of the understanding, the proposition, “Every change must have a cause,” will amply serve our purpose. In the latter case, indeed, the conception of a cause so plainly involves the conception of a necessity of connection with an effect, and of a strict universality of the law, that the very notion of a cause would entirely disappear, were we to derive it, like Hume, from a frequent association of what happens with that which precedes; and the habit thence originating of connecting representations—the necessity inherent in the judgement being therefore merely subjective. Besides, without seeking for such examples of principles existing a priori in cognition, we might easily show that such principles are the indispensable basis of the possibility of experience itself, and consequently prove their existence a priori. For whence could our experience itself acquire certainty, if all the rules on which it depends were themselves empirical, and consequently fortuitous? No one, therefore, can admit the validity of the use of such rules as first principles. But, for the present, we may content ourselves with having established the fact, that we do possess and exercise a faculty of pure a priori cognition; and, secondly, with having pointed out the proper tests of such cognition, namely, universality and necessity.

Not only in judgements, however, but even in conceptions, is an a priori origin manifest. For example, if we take away by degrees from our conceptions of a body all that can be referred to mere sensuous experience—colour, hardness or softness, weight, even impenetrability—the body will then vanish; but the space which it occupied still remains, and this it is utterly impossible to annihilate in thought. Again, if we take away, in like manner, from our empirical conception of any object, corporeal or incorporeal, all properties which mere experience has taught us to connect with it, still we cannot think away those through which we cogitate it as substance, or adhering to substance, although our conception of substance is more determined than that of an object. Compelled, therefore, by that necessity with which the conception of substance forces itself upon us, we must confess that it has its seat in our faculty of cognition a priori.
III. Philosophy stands in need of a Science which shall Determine the Possibility, Principles, and Extent of Human Knowledge “a priori”

Of far more importance than all that has been above said, is the consideration that certain of our cognitions rise completely above the sphere of all possible experience, and by means of conceptions, to which there exists in the whole extent of experience no corresponding object, seem to extend the range of our judgements beyond its bounds. And just in this transcendental or supersensible sphere, where experience affords us neither instruction nor guidance, lie the investigations of reason, which, on account of their importance, we consider far preferable to, and as having a far more elevated aim than, all that the understanding can achieve within the sphere of sensuous phenomena. So high a value do we set upon these investigations, that even at the risk of error, we persist in following them out, and permit neither doubt nor disregard nor indifference to restrain us from the pursuit. These unavoidable problems of mere pure reason are God, freedom (of will), and immortality. The science which, with all its preliminaries, has for its especial object the solution of these problems is named metaphysics—a science which is at the very outset dogmatical, that is, it confidently takes upon itself the execution of this task without any previous investigation of the ability or inability of reason for such an undertaking.

Now the safe ground of experience being thus abandoned, it seems nevertheless natural that we should hesitate to erect a building with the cognitions we possess, without knowing whence they come, and on the strength of principles, the origin of which is undiscovered. Instead of thus trying to build without a foundation, it is rather to be expected that we should long ago have put the question, how the understanding can arrive at these a priori cognitions, and what is the extent, validity, and worth which they may possess? We say, “This is natural enough,” meaning by the word natural, that which is consistent with a just and reasonable way of thinking; but if we understand by the term, that which usually happens, nothing indeed could be more natural and more comprehensible than that this investigation should be left long unattempted. For one part of our pure knowledge, the science of mathematics, has been long firmly established, and thus leads us to form flattering expectations with regard to others, though these may be of quite a different nature. Besides, when we get beyond the bounds of experience, we are of course safe from opposition in that quarter; and the charm of widening the range of our knowledge is so great that, unless we are brought to a standstill by some evident contradiction, we hurry on undoubtingly in our course. This, however, may be avoided, if we are sufficiently cautious in the construction of our fictions, which are not the less fictions on that account.

Mathematical science affords us a brilliant example, how far, independently of all experience, we may carry our a priori knowledge. It is true that the mathematician occupies himself with objects and cognitions only in so far as they can be represented by means of intuition. But this circumstance is easily overlooked, because the said intuition can itself be given a priori, and therefore is hardly to be distinguished from a mere pure conception. Deceived by such a proof of the power of reason, we can perceive no limits to the extension of our knowledge. The light dove cleaving in free flight the thin air, whose resistance it feels, might imagine that her movements would be far more free and rapid in airless space. Just in the same way did Plato, abandoning the world of sense because of the narrow limits it sets to the understanding, venture upon the wings of ideas beyond it, into the void space of pure intellect. He did not reflect that he made no real progress by all his efforts; for he met with no resistance which might serve him for a support, as it were, whereon to rest, and on which he might apply his powers, in order to let the intellect acquire momentum for its progress. It is, indeed, the common fate of human reason in speculation, to finish the imposing edifice of thought as rapidly as possible, and then for the first time to begin to examine whether the foundation is a solid one or no. Arrived at this point, all sorts
of excuses are sought after, in order to console us for its want of stability, or rather, indeed, to enable Us to dispense altogether with so late and dangerous an investigation. But what frees us during the process of building from all apprehension or suspicion, and flatters us into the belief of its solidity, is this. A great part, perhaps the greatest part, of the business of our reason consists in the analysis of the conceptions which we already possess of objects. By this means we gain a multitude of cognitions, which although really nothing more than elucidations or explanations of that which (though in a confused manner) was already thought in our conceptions, are, at least in respect of their form, prized as new introspections; whilst, so far as regards their matter or content, we have really made no addition to our conceptions, but only disinvolved them. But as this process does furnish a real a priori knowledge, which has a sure progress and useful results, reason, deceived by this, slips in, without being itself aware of it, assertions of a quite different kind; in which, to given conceptions it adds others, a priori indeed, but entirely foreign to them, without our knowing how it arrives at these, and, indeed, without such a question ever suggesting itself. I shall therefore at once proceed to examine the difference between these two modes of knowledge.

IV. Of the Difference Between Analytical and Synthetical Judgements.
In all judgements wherein the relation of a subject to the predicate is cogitated (I mention affirmative judgements only here; the application to negative will be very easy), this relation is possible in two different ways. Either the predicate B belongs to the subject A, as somewhat which is contained (though covertly) in the conception A; or the predicate B lies completely out of the conception A, although it stands in connection with it. In the first instance, I term the judgement analytical, in the second, synthetical. Analytical judgements (affirmative) are therefore those in which the connection of the predicate with the subject is cogitated through identity; those in which this connection is cogitated without identity, are called synthetical judgements. The former may be called explicative, the latter augmentative judgements; because the former add in the predicate nothing to the conception of the subject, but only analyse it into its constituent conceptions, which were thought already in the subject, although in a confused manner; the latter add to our conceptions of the subject a predicate which was not contained in it, and which no analysis could ever have discovered therein. For example, when I say, “All bodies are extended,” this is an analytical judgement. For I need not go beyond the conception of body in order to find extension connected with it, but merely analyse the conception, that is, become conscious of the manifold properties which I think in that conception, in order to discover this predicate in it: it is therefore an analytical judgement. On the other hand, when I say, “All bodies are heavy,” the predicate is something totally different from that which I think in the mere conception of a body. By the addition of such a predicate, therefore, it becomes a synthetical judgement.

Judgements of experience, as such, are always synthetical. For it would be absurd to think of grounding an analytical judgement on experience, because in forming such a judgement I need not go out of the sphere of my conceptions, and therefore recourse to the testimony of experience is quite unnecessary. That “bodies are extended” is not an empirical judgement, but a proposition which stands firm a priori. For before addressing myself to experience, I already have in my conception all the requisite conditions for the judgement, and I have only to extract the predicate from the conception, according to the principle of contradiction, and thereby at the same time become conscious of the necessity of the judgement, a necessity which I could never learn from experience. On the other hand, though at first I do not at all include the predicate of weight in my conception of body in general, that conception still
indicates an object of experience, a part of the totality of experience, to which I can still add other parts; and this I do when I recognize by observation that bodies are heavy. I can cognize beforehand by analysis the conception of body through the characteristics of extension, impenetrability, shape, etc., all which are cogitated in this conception. But now I extend my knowledge, and looking back on experience from which I had derived this conception of body, I find weight at all times connected with the above characteristics, and therefore I synthetically add to my conceptions this as a predicate, and say, “All bodies are heavy.” Thus it is experience upon which rests the possibility of the synthesis of the predicate of weight with the conception of body, because both conceptions, although the one is not contained in the other, still belong to one another (only contingently, however), as parts of a whole, namely, of experience, which is itself a synthesis of intuitions.

But to synthetical judgements a priori, such aid is entirely wanting. If I go out of and beyond the conception A, in order to recognize another B as connected with it, what foundation have I to rest on, whereby to render the synthesis possible? I have here no longer the advantage of looking out in the sphere of experience for what I want. Let us take, for example, the proposition, “Everything that happens has a cause.” In the conception of “something that happens,” I indeed think an existence which a certain time antecedes, and from this I can derive analytical judgements. But the conception of a cause lies quite out of the above conception, and indicates something entirely different from “that which happens,” and is consequently not contained in that conception. How then am I able to assert concerning the general conception—“that which happens”—something entirely different from that conception, and to recognize the conception of cause although not contained in it, yet as belonging to it, and even necessarily? what is here the unknown = X, upon which the understanding rests when it believes it has found, out of the conception A a foreign predicate B, which it nevertheless considers to be connected with it? It cannot be experience, because the principle adduced annexes the two representations, cause and effect, to the representation existence, not only with universality, which experience cannot give, but also with the expression of necessity, therefore completely a priori and from pure conceptions. Upon such synthetical, that is augmentative propositions, depends the whole aim of our speculative knowledge a priori; for although analytical judgements are indeed highly important and necessary, they are so, only to arrive at that clearness of conceptions which is requisite for a sure and extended synthesis, and this alone is a real acquisition.

V. In all Theoretical Sciences of Reason, Synthetical Judgements “a priori” are contained as Principles. 1. Mathematical judgements are always synthetical. Hitherto this fact, though incontestably true and very important in its consequences, seems to have escaped the analysts of the human mind, nay, to be in complete opposition to all their conjectures. For as it was found that mathematical conclusions all proceed according to the principle of contradiction (which the nature of every apodeictic certainty requires), people became persuaded that the fundamental principles of the science also were recognized and admitted in the same way. But the notion is fallacious; for although a synthetical proposition can certainly be discerned by means of the principle of contradiction, this is possible only when another synthetical proposition precedes, from which the latter is deduced, but never of itself.

Before all, be it observed, that proper mathematical propositions are always judgements a priori, and not empirical, because they carry along with them the conception of necessity, which cannot be given by
experience. If this be demurred to, it matters not; I will then limit my assertion to pure mathematics, the very conception of which implies that it consists of knowledge altogether non-empirical and a priori.

We might, indeed at first suppose that the proposition $7 + 5 = 12$ is a merely analytical proposition, following (according to the principle of contradiction) from the conception of a sum of seven and five. But if we regard it more narrowly, we find that our conception of the sum of seven and five contains nothing more than the uniting of both sums into one, whereby it cannot at all be cogitated what this single number is which embraces both. The conception of twelve is by no means obtained by merely cogitating the union of seven and five; and we may analyse our conception of such a possible sum as long as we will, still we shall never discover in it the notion of twelve. We must go beyond these conceptions, and have recourse to an intuition which corresponds to one of the two—our five fingers, for example, or like Segner in his Arithmetic five points, and so by degrees, add the units contained in the five given in the intuition, to the conception of seven. For I first take the number 7, and, for the conception of 5 calling in the aid of the fingers of my hand as objects of intuition, I add the units, which I before took together to make up the number 5, gradually now by means of the material image my hand, to the number 7, and by this process, I at length see the number 12 arise. That 7 should be added to 5, I have certainly cogitated in my conception of a sum = 7 + 5, but not that this sum was equal to 12. Arithmetical propositions are therefore always synthetical, of which we may become more clearly convinced by trying large numbers. For it will thus become quite evident that, turn and twist our conceptions as we may, it is impossible, without having recourse to intuition, to arrive at the sum total or product by means of the mere analysis of our conceptions. Just as little is any principle of pure geometry analytical. “A straight line between two points is the shortest,” is a synthetical proposition. For my conception of straight contains no notion of quantity, but is merely qualitative. The conception of the shortest is therefore fore wholly an addition, and by no analysis can it be extracted from our conception of a straight line. Intuition must therefore here lend its aid, by means of which, and thus only, our synthesis is possible.

Some few principles preposited by geometricians are, indeed, really analytical, and depend on the principle of contradiction. They serve, however, like identical propositions, as links in the chain of method, not as principles—for example, $a = a$, the whole is equal to itself, or $(a+b) = a$, the whole is greater than its part. And yet even these principles themselves, though they derive their validity from pure conceptions, are only admitted in mathematics because they can be presented in intuition. What causes us here commonly to believe that the predicate of such apodeictic judgements is already contained in our conception, and that the judgement is therefore analytical, is merely the equivocal nature of the expression. We must join in thought a certain predicate to a given conception, and the necessity cleaves already to the conception. But the question is, not what we must join in thought to the given conception, but what we really think therein, though only obscurely, and then it becomes manifest that the predicate pertains to these conceptions, necessarily indeed, yet not as thought in the conception itself, but by virtue of an intuition, which must be added to the conception.

2. The science of natural philosophy (physics) contains in itself synthetical judgements a priori, as principles. I shall adduce two propositions. For instance, the proposition, “In all changes of the material world, the quantity of matter remains unchanged”; or, that, “In all communication of motion, action and reaction must always be equal.” In both of these, not only is the necessity, and therefore their origin a priori clear, but also that they are synthetical propositions. For in the conception of matter, I do not cogitate its permanency, but merely its presence in space, which it fills. I therefore really go out of and beyond the conception of matter, in order to think on to it something a priori, which I did not think in it.
The proposition is therefore not analytical, but synthetical, and nevertheless conceived a priori; and so it is with regard to the other propositions of the pure part of natural philosophy.

3. As to metaphysics, even if we look upon it merely as an attempted science, yet, from the nature of human reason, an indispensable one, we find that it must contain synthetical propositions a priori. It is not merely the duty of metaphysics to dissect, and thereby analytically to illustrate the conceptions which we form a priori of things; but we seek to widen the range of our a priori knowledge. For this purpose, we must avail ourselves of such principles as add something to the original conception—something not identical with, nor contained in it, and by means of synthetical judgements a priori, leave far behind us the limits of experience; for example, in the proposition, “the world must have a beginning,” and such like. Thus metaphysics, according to the proper aim of the science, consists merely of synthetical propositions a priori.